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INFLUENCE OF RADIATION ON THE DEGENERATION OF ISOTROPIC TURBULENCE IN HIGH-TEMPERATURE MEDIA

I. A. Vatutin, B. A. Kolovandin, and O. G. Martynenko

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It is shown that radiation exerts a distinct influence on the degeneration of isotropic turbulence depending on the vortex size in conformity with the formula obtained for radiant thermal diffusivity.

Sufficiently many journal articles and monographs are devoted to the investigation of radiation interaction with a substance and to questions of the dynamics of a radiating gas. However, the interaction between radiation and turbulence during motion of high-temperature media has been inadequately studied. Meanwhile, as has been shown in [1, 2], this interaction is substantial for a number of problems of practical importance. The influence of radiation on the structure of degenerating isotropic turbulence in compressible high-temperature gases is examined below.

It can be shown [2, 3] that at temperatures to many thousands of degrees the magnitude of the total volume density of radiation for not very rarefied media is small compared to the volume energy density of the particle thermal motion in the medium. This also refers to the so-called radiant pressure which is small compared to the pressure caused by particle motion in a medium under the conditions mentioned. At the same time, because of the high velocity of radiation propagation the radiation energy transfer can be substantially greater than the energy transfer during motion of the medium or motion of the particles in the medium. We shall later limit ourselves to the case when the equation of state of a ideal gas is approximately valid, and the specific heats c_p and c_V are separately constant, i.e., are independent of the temperature.

Under the constraints mentioned, the continuity, motion, and energy equations have the form [2, 4]

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.

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$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_k}(\rho u_k) = 0, \tag{1}$$

$$\rho\left(\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k}\right) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial^2 u_i}{\partial x_k^2} + \frac{1}{3}(\mu + \xi)\frac{\partial^2 u_i}{\partial x_k \partial x_i}$$
$$+\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3}\frac{\partial u_e}{\partial x_e}\delta_{ik}\right)\frac{\partial \mu}{\partial x_k} + \frac{\partial u_e}{\partial x_e}\frac{\partial \xi}{\partial x_i}, \tag{2}$$

$$\rho c_V\left(\frac{\partial T}{\partial t} + u_k \frac{\partial T}{\partial x_k}\right) = -p\frac{\partial u_k}{\partial x_k} + \sigma_{ik}\frac{\partial u_i}{\partial x_k} + \frac{\partial}{\partial x_k}\left(k\frac{\partial T}{\partial x_k}\right) - \frac{\partial}{\partial x_k}(q_{\text{луч}}), \tag{3}$$

where μ is the coefficient of viscosity; ξ, second coefficient of volume viscosity; k, coefficient of heat conductivity, and

$$\sigma_{ik} = \mu\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} - \frac{2}{3}\frac{\partial u_e}{\partial x_e}\delta_{ik}\right) + \xi\frac{\partial u_e}{\partial x_e}\delta_{ik}.$$

The radiant heat flux is

$$q_{\text{rad}} = \int\limits_{\nu=0}^{\infty}\int\limits_{\Omega} \mathbf{I}_\nu d\Omega d\nu, \tag{4}$$

where the spectral radiation intensity I_γ equals the quantity of energy transferred by radiation in some direction i referred to unit time, to a unit perpendicular to the area, to a unit solid angle Ω, and a unit frequency band.

The change in radiation intensity along a ray is determined by the processes of radiation, absorption, and scattering, as well as by the change in the volume density of radiation in time. Taking into account that the role of the radiation scattering process is small for high-temperature media [2], we write the radiation transport equation for locally equilibrium media in the form

$$\frac{\partial I_\nu}{\partial s} = k_\nu(B_\nu - I_\nu), \tag{5}$$

where the spectral intensity of the black radiation B_ν is determined by the Planck formula

$$B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{\exp\left(\frac{h\nu}{KT}\right) - 1},$$

and k_ν is the difference between the true coefficient of absorption and the coefficient of induced radiation.

A div q_{rad} enters into (3). As has been indicated in [2], this quantity can be found for a known distribution I_ν without using (4), since it is the quantity of radiation energy "being generated" (because of the difference between radiation and absorption) per unit volume per unit time, and therefore, is equal to the integral of the right side of (5) with respect to the frequency and solid angle (in all directions):

$$\operatorname{div} q_{\text{rad}} = \int\limits_{\nu}^{\infty}\int\limits_{\Omega} k_\nu(B_\nu - I_\nu)\,d\nu d\Omega = \int\limits_{\nu}^{\infty}\left\{4\pi k_\nu B_\nu - \int\limits_{\Omega} k_\nu I_\nu d\Omega\right\}d\nu. \tag{6}$$

Let us consider the two limit cases of large and small optical densities of the medium. If the condition $Lk_\nu \gg 1$ (L is the dimension of the turbulent vortices) is satisfied for the radiation of all frequencies ν essential in energy respects, then the so-called radiant heat-conduction approximation [2] is applicable, and

$$q_{\text{rad}} = -k_{\text{rad}}\nabla T, \tag{7}$$

where $k_{\text{rad}} = \frac{4\pi}{3}\int\limits_0^\infty \frac{1}{k_\nu}\frac{\partial B_\nu}{\partial T}d\nu$ is the coefficient of radiant heat conductivity which depends on the local parameters of the medium, and can be added with the coefficient of heat conductivity determined by the thermal motion of the particles.

If $k_\nu L \ll 1$, then the pulsation in the quantity I_ν will be quite weakly correlated to the local pulsations of the thermophysical quantities since the quantity I_ν is determined by the temperature distribution in a length on the order of $1/k_\nu \gg L$, and hence, is later neglected by the correlations $\langle I_\nu' D' \rangle$, where D is any local thermophysical quantity. The viscosity and heat conductivity coefficients dependent on hydrodynamic quantities (primarily on the temperature which makes obtaining a closed system of equations for the correlation functions difficult) enter into the equation; moreover, the derivatives of the fundamental fields (e.g., $\partial u_i/\partial x_k$) must be taken as new unknowns and a number of moments of nonlinear functions of the initial fields that occur in connection with the nonlinearity of the equation of state must still enter into the analysis. Consequently, an awkward system is obtained that contains very much more unknowns than equations, which is quite difficult to utilize. Hence, following mainly [5, 6] we examine the simplest case of very weak turbulence described by the linearized hydromechanics equations, i.e., when the turbulent fluctuations are so slight that the third moments of all the hydrodynamic fields are negligible in comparison with the appropriate second moments. In other words, we assume that the turbulence under consideration has already reached the "concluding period of degeneration."

Since fluctuations of an arbitrary function of the hydrodynamic fields are represented in a linear approximation as a linear combination (with constant coefficients) of the initial field fluctuations, then any change of variables in the hydromechanics equations in this approximation will reduce to a trivial linear transformation of the system of equations for the correlation functions. To use the method already developed to investigate the degeneration of isotropic turbulence in compressible nonradiating media [3, 6, 7], we use the quantities u_i ($i = 1, 2, 3$), the dimensionless pressure $P = p/\gamma P_0$, and the entropy divided by c_p, or $S = s/c_p$, as the fundamental variables. Starting from the above, we easily obtain a linearized system of equations in the fluctuations u_i, S, and P from (1)-(3) (by using Taylor series expansions).

To obtain a system of equations in the correlation functions of the fields mentioned, each of the equations containing the derivative $(\partial/\partial t)\alpha(x, t)$ in the left side (where $\alpha = u_i'$, P' or S') must be multiplied by $\beta(x', t) = \beta(x + r, t)$ (where β is one of the same five quantities), the equality obtained must be added to the equation for $(\partial/\partial t)\beta(x, t)$, multiplied by $\alpha(x, t)$, and the result must be averaged. In the case of homogeneous turbulence in which all the correlation functions $\langle \alpha(x, t)\beta(x', t) \rangle$ depend only on $\mathbf{r} = (\mathbf{x}' - \mathbf{x})$ and on t, $\partial/\partial x_i$ should then be replaced everywhere in the equalities obtained by $(-\partial/\partial r_i)$ and $\partial/\partial x_j'$ by $\partial/\partial r_j$. The system of equations thus obtained is sufficiently complicated and hardly discernable physically. Hence, it is more convenient to go from the very beginning over to spectral equations. Subjecting all the equations obtained by the above-mentioned method to Fourier transformation and expressing the tensor $F_{ij}(k, t)$ and the vectors $F_{ip}(k, t)$ and $F_{is}(k, t)$ in terms of the scalar functions $F_{LL}(k, t)$, $F_{NN}(k, t)$, $F_{Lp}(k, t)$, and $F_{Ls}(k, t)$ by using the known isotropic turbulence relationships [6], we consequently arrive at the following system of ordinary differential equations for the spectral functions describing the evolution of weak isotropic turbulence in a compressible radiating gas

$$\frac{\partial}{\partial t} F_{NN} = -2\tilde{\nu} k^2 F_{NN},$$

$$\frac{\partial}{\partial t} F_{LL} = -2\tilde{\nu}_1 k^2 F_{LL} + 2a_0^2 k F_{Lp},$$

$$\frac{\partial F_{Lp}}{\partial t} = -kF_{LL} - [\tilde{\nu}_1 + (\gamma - 1)(\chi + \chi_{\mathrm{rad}})] k^2 F_{Lp} - (\chi + \chi_{\mathrm{rad}}) k^2 F_{Ls} + a_0^2 k F_{pp},$$

$$\frac{\partial}{\partial t} F_{Ls} = -(\gamma - 1)(\chi + \chi_{\mathrm{rad}}) k^2 F_{Lp} - [\tilde{\nu}_1 + (\chi + \chi_{\mathrm{rad}})] k^2 F_{Ls} + a_0^2 k F_{ps},$$

$$\frac{\partial}{\partial t} F_{pp} = -2kF_{Lp} - 2(\gamma_1 - 1)(\chi + \chi_{\mathrm{rad}}) k^2 F_{pp} - 2(\chi + \chi_{\mathrm{rad}}) k^2 F_{ps},$$

$$\frac{\partial}{\partial t} F_{ps} = -kF_{Ls} - (\gamma - 1)(\chi + \chi_{\mathrm{rad}}) k^2 F_{pp} - \gamma(\chi + \chi_{\mathrm{rad}}) k^2 F_{ps} - (\chi + \chi_{\mathrm{rad}}) k^2 F_{ss},$$

$$\frac{\partial}{\partial t} F_{ss} = -2(\gamma - 1)(\chi + \chi_{\mathrm{rad}}) k^2 F_{ps} - 2(\chi + \chi_{\mathrm{rad}}) k^2 F_{ss}, \qquad (8)$$

where $\tilde{\nu} = \frac{\mu}{\rho}$; $\tilde{\nu}_1 = \frac{4}{3}\frac{\mu}{\rho} + \frac{\xi}{\rho}$; $\gamma = \frac{c_p}{c_V}$; $a_0 = \sqrt{\gamma \cdot \frac{P_0}{\rho_0}}$ is the speed of sound in the unperturbed medium; k, wave number equal to $2\pi/L$ (L is the size of the turbulent vortices); $\chi = k/\rho_0 c_p$, F_{NN} and F_{LL}, spectral densities corresponding to the transverse and longitudinal correlation functions of the velocity fields, and the remaining $F_{\alpha\beta}$ have analogous meaning.

The coefficient χ_{rad} enters the equations in the system (8). If $k_\nu L \gg 1$, then

$$\chi_{rad} = \frac{k_{rad}}{\rho_\partial c_p} = \frac{\frac{4\pi}{3}\int\limits_0^\infty \frac{1}{k_\nu}\frac{\partial B_\nu}{\partial T}d\nu}{\rho_0 c_p}. \tag{9}$$

Under the condition $k_\nu L \ll 1$

$$\chi_{rad} = \frac{f}{\rho_0 c_p k^2}, \tag{10}$$

where

$$f = 4\pi\frac{\partial}{\partial T}\int\limits_\nu^\infty k_\nu B_\nu d\nu - \int\limits_\nu^\infty\left\{\frac{\partial k_\nu}{\partial T}\int\limits_\Omega <I_\nu> d\Omega\right\}d\nu. \tag{11}$$

Here k_ν, B_ν, $\partial k_\nu/\partial T$ and $\partial B_\nu/\partial T$ should be evaluated in (9)-(11) for $T = <T>$. By using (5) it is easy to show that $<I_\nu> \approx <B_\nu>$ in a linear approximation. Hence, the second term on the right in (11) is a thermophysical property and equals $4\pi\int\limits_\nu^\infty B_\nu \frac{\partial k_\nu}{\partial T}d\nu$ for $T = <T>$.

Therefore, the influence of radiation on turbulence is that the ordinary thermal diffusivity χ should be replaced by some effective thermal diffusivity

$$\chi_{ef} = \chi + \chi_{rad}, \tag{12}$$

where χ_{rad} is determined by (9) and (10), respectively, for optically thick and optically thin media.

If $\chi_{rad} = 0$, the solution of system (8) is elucidated in [6], and the general solution for $\chi_{rad} \neq 0$ will naturally have the same form. However, in obtaining simple limit expressions, e.g., as $k \to 0$, and also when $\chi_{rad} \gg \chi$, substantial differences appear. Consequently, we briefly describe the method of solution and we write the limit expressions for the spectral and correlation functions, when χ_{rad} and χ are commensurate in order of magnitude, and when $\chi_{rad} \gg \chi$ as $k \to 0$.

The solution of the first equation in system (8) is sought separately, and it is examined in [6]. Hence, it is sufficient for us to consider the six remaining equations which form a system of ordinary linear differential equations with constant coefficients for fixed k. To solve this system, a characteristic equation determining the particular solutions proportional to $\exp(\omega t)$ is formed first. Expanding the sixth-order determinant giving this characteristic equation, we obtain a sixth-power algebraic equation that is reducible, since it can be shown that its left side equals the product of two third-power polynomials. Using the relationships between the roots and coefficients of the polynomials, we see easily that the six roots of the characteristic equation equal all the possible sums of two roots (perhaps repeated) of the following cubic equation

$$\omega^3 + (\tilde{\nu}_1 + \gamma\chi_{ef})k^2\omega^2 + \left(\gamma\tilde{\nu}_1\chi_{ef} + \frac{a_0^2}{k^2}\right)k^4\omega + \chi_{ef}a_0^2k^4 = 0. \tag{13}$$

If the time t is replaced by the dimensionless time $\tau = a_0 kt$, then ω goes over into the dimensionless quantity $\lambda = \omega/a_0 k$, and (13) into the equation

$$\lambda^3 + \frac{(\tilde{\nu}_1 + \gamma\chi_{ef})k}{a_0}\lambda^2 + \left(1 + \frac{\gamma\tilde{\nu}_1\chi_{ef}k^2}{a_0^2}\right)\lambda + \frac{\chi_{ef}k}{a_0} = 0. \tag{14}$$

Solving (14) by the Cardano method [8], say, the eigenvalues can be found, and then the eigenvectors, and therefore, the general solution for the spectral functions. Hence

$$F_{NN}(k, t) = c_0(k)\exp(-2\tilde{\nu}k^2t), \tag{15}$$

while the remaining six functions of the system (8) are expressed as follows

$$F_{\alpha\beta}(k, t) = c_1(k)b_{\alpha\beta}^{(1)}\exp(2\omega_1 t) + c_2(k)b_{\alpha\beta}^{(2)}\exp(2\omega_2 t) + c_3(k)b_{\alpha\beta}^{(3)}\exp[(\omega_1+\omega_2)t]$$
$$+ c_4(k)b_{\alpha\beta}^{(4)}\exp[(\omega_1+\omega_3)t] + c_5(k)b_{\alpha\beta}^{(5)}\exp[(\omega_2+\omega_3)t] + c_6(k)b_{\alpha\beta}^{(6)}\exp(2\omega_3 t), \tag{16}$$

where ω_1, ω_2, ω_3 are the roots of (13); $c_0(k)$, $c_1(k)$, ..., $c_6(k)$ are seven functions of k such that $c_2(k) = [c_1(k)]^*$, $c_5(k) = [c_4(k)]^*$ (the sign $[]^*$ denotes the complex conjugate), while the remaining $c_i(k)$ are real. The coefficients $b_{\alpha\beta}^{(1)}, \ldots, b_{\alpha\beta}^{(6)}$ are proportional to the values of the corresponding cofactors of the characteristic determinant of system (8). Therefore, solutions (15) and (16) of system (8) depend on seven arbitrary real functions defined by the initial values of the corresponding spectral functions.

A further simplification of the results obtained can be achieved if we note that (14) and the system of equations (8) contain dimensionless constants having a different order of magnitude. In fact, all the coefficients in (14) can be expressed in terms of three dimensionless quantities

$$\delta_1 = \frac{\tilde{v}_1 k}{a_0}, \quad \tilde{\Pr} = \frac{\tilde{v}_1}{\chi_{ef}}, \quad \gamma = \frac{c_p}{c_V}. \tag{17}$$

The parameter $\delta = \tilde{\nu}k/\omega_0$ is of the same order as the ratio l/L between the gas mean free path l and the dimension of the turbulent vortex $L = 2\pi/k$ (since $\tilde{\nu} \cong vl$, where v is the mean velocity of thermal molecular motion, $a_0 \sim v$, $k \cong 1/L$). Hence, in all cases when the gas motions can be described by using the ordinary hydrodynamic equations $\delta \ll 1$; in particular, for air under normal conditions $\tilde{\nu}/a_0 \approx 0.5 \cdot 10^{-5}$ cm, i.e., even if $L \approx 1$ mm, then $\delta < 10^{-4}$. Then $\delta_1 = (\tilde{\nu}_1/\tilde{\nu})\delta$ also has the same order of magnitude.

Let us write (14) with (17) taken into account:

$$\lambda^3 + \left(1 + \frac{\gamma}{\tilde{\Pr}}\right)\delta_1\lambda^2 + \left(1 + \frac{\gamma}{\tilde{\Pr}}\delta_1^2\right)\lambda + \frac{\delta_1}{\tilde{\Pr}}. \tag{18}$$

Therefore, if χ_{rad} is less than or commensurate with χ in order of magnitude, then $\tilde{\Pr} \approx 1$ and the three roots of (18) are determined easily to the accuracy of terms of the order δ_1 inclusive by using the usual power-series method. We consequently obtain

$$\begin{aligned}
\lambda_1 &= i - \frac{(\tilde{\Pr} + \gamma - 1)}{2\tilde{\Pr}}\delta_1, \\
\lambda_2 &= -i - \frac{(\tilde{\Pr} + \gamma - 1)}{2\tilde{\Pr}}\delta_1, \\
\lambda_3 &= -\frac{\delta_1}{\tilde{\Pr}},
\end{aligned} \tag{19}$$

where $\tilde{\Pr} = \tilde{\nu}_1/(\chi + \chi_{rad})$.

Let us note that for the case when $k_\nu L \ll 1$ and χ_{rad} is defined by (10), we limit ourselves to definite values of the wave number k since χ_{rad} grows strongly but the number $\tilde{\Pr}$ decreases as $k \to 0$.

To δ_1 accuracy the solutions of the system (8) are determined by the formulas

$$\begin{aligned}
F_{NN}(k, t) &= c_0(k)\exp(-2\tilde{v}k^2 t), \\
F_{LL}(k, t) &= c_1(k)a_0^2\exp(2\omega_1 t) + c_2(k)a_0^2\exp(2\omega_2 t) + c_3(k)a_0^2\exp[(\omega_1+\omega_2)t], \\
F_{Lp}(k, t) &= ic_1(k)a_0\exp(2\omega_1 t) - ic_2(k)a_0\exp(2\omega_2 t), \\
F_{Ls}(k, t) &= c_4(k)a_0\exp[(\omega_1+\omega_3)t] + c_5(k)a_0\exp[(\omega_2+\omega_3)t], \\
F_{pp}(k, t) &= -c_1(k)\exp(2\omega_1 t) - c_2(k)\exp(2\omega_2 t) + c_3(k)\exp[(\omega_1+\omega_2)t], \\
F_{ps}(k, t) &= ic_4(k)\exp[(\omega_1+\omega_3)t] - ic_5(k)\exp[(\omega_2+\omega_3)t], \\
F_{ss}(k, t) &= c_6(k)\exp(2\omega_3 t),
\end{aligned} \tag{20}$$

where $i^2 = -1$; $c_0(k), c_1(k), \ldots, c_6(k)$ satisfy the same conditions as in (16), $\omega_i = a_0 k\lambda_i$ ($i = 1, 2, 3$), and the λ_i have the same meaning as in (19).

If $\chi_{rad} \gg \chi$ (in case $k_\gamma L \ll 1$ this will be satisfied always as $k \to 0$), the χ in the exact solution of (14) by the Cardano method [8] should be neglected in comparison to χ_{rad}, and the principal terms retained as $k \to 0$. We then find

$$\begin{cases} \lambda_1 = \lambda_2 = 0, \\ \lambda_3 = -\gamma\dfrac{\delta_1}{\tilde{\Pr}} \end{cases} \quad \text{or} \quad \begin{cases} \omega_1 = \omega_2 = 0, \\ \omega_3 = -a_0 k\gamma\dfrac{\delta_1}{\tilde{\Pr}} = \gamma\dfrac{f}{\rho_0 c_p}. \end{cases} \tag{21}$$

In this case we obtain the following expressions in place of (20):

$$F_{NN}(k,\ t)=c_0(k)\exp(-2\tilde{\nu}k^2t),$$

$$F_{LL}(k,t)=c_1(k)a_0^2\exp(2\omega_1t)+c_5(k)a_0^2\exp[(\omega_1+\omega_3)t]+c_2(k)a_0^2\exp(2\omega_2t),$$

$$F_{Lp}(k,\ t)=c_1(k)a_0\exp(2\omega_1t)+c_5(k)a_0\exp[(\omega_1+\omega_3)t],$$

$$F_{pp}(k,\ t)=\frac{1}{(\gamma-1)}c_4(k)\exp[(\omega_1+\omega_2)t]+c_6(k)\exp[(\omega_2+\omega_3)t]+c_3(k)\exp(2\omega_3t),\tag{22}$$

$$F_{Ls}(k,\ t)=-(\gamma-1)c_1(k)a_0\exp(2\omega_1t)+c_5(k)a_0\exp[(\omega_1+\omega_3)t],$$

$$F_{ps}(k,\ t)=c_4(k)\exp[(\omega_1+\omega_2)t]+\frac{(2-\gamma)}{2}c_6(k)\exp[(\omega_2+\omega_3)t]+c_3(k)\exp(2\omega_3t),$$

$$F_{ss}(k,\ t)=-(\gamma-1)c_4(k)\exp[(\omega_1+\omega_2)t]-(\gamma-1)c_6(k)\exp[(\omega_2+\omega_3)t]+c_3(k)\exp(2\omega_3t).$$

where all the $c_0(k)$, $c_1(k)$, ..., $c_6(k)$ are real and determined by the initial values of the spectral functions.

For known initial values of the spectral functions we can always obtain expressions for the corresponding correlation functions by means of the formulas [6]:

$$B(r)=4\pi\int_0^\infty\frac{\sin kr}{kr}F(k)k^2dk,$$

$$B_{L\theta}=4\pi\int_0^\infty\left\{-\frac{\cos kr}{kr}+\frac{\sin kr}{(kr)^2}\right\}F_{L\theta}(k)k^2dk,\tag{23}$$

$$B_{LL}(r)=4\pi\int_0^\infty\left\{\frac{\sin kr}{kr}+2\frac{\cos kr}{(kr)^2}-2\frac{\sin kr}{(kr)^3}\right\}F_{LL}(k)k^2dk+4\pi\int_0^\infty\left\{-2\frac{\cos kr}{(kr)^2}+2\frac{\sin kr}{(kr)^3}\right\}F_{NN}(k)k^2dk,$$

$$B_{NN}(r)=4\pi\int_0^\infty\left\{-\frac{\cos kr}{(kr)^2}+\frac{\sin kr}{(kr)^3}\right\}F_{LL}(k)k^2dk+4\pi\int_0^\infty\left\{\frac{\sin kr}{kr}+\frac{\cos kr}{(kr)^2}-\frac{\sin kr}{(kr)^3}\right\}F_{NN}(k)k^2dk,$$

where $\theta = P$ or S and $B(r)$ is a correlation function of scalar quantities.

We present here the expression of just certain correlation functions for the case $k_\nu L \gg 1$ and when χ_{rad} is less than or comparable to χ in order of magnitude. In this case, all six exponents $\omega = 2\omega_1$, $2\omega_2$, ..., $2\omega_3$ have negative real part for any $k > 0$ because of (19). Hence, parts of the integrals (23), taken from $k = \varepsilon$ to $k = \infty$ for any $\varepsilon > 0$ will yield only the exponentially damped component in the expression for the appropriate correlation function. Therefore, the asymptotic behavior of all the correlation functions will be determined only by values of the integrals, taken between 0 and ε, in the right sides of (23) as $k\to\infty$, and will depend only on the behavior of the corresponding spectral density $F_{\alpha\beta}(k,\ t)$ in the neighborhood of the point $k = 0$. In investigating the principal term of the spectral functions around $k = 0$, we can use the simplified formulas (20) fully since terms on the order of $\delta_1 = \tilde{\nu}_1 k/a_0$, which vanish as $k\to 0$, have been omitted. Moreover, for the same reason we can now use the simplified formulas (19) to determine the exponents $\omega_i = a_0 k\lambda_i$, and take the coefficients $c_0(k)$, $c_1(k)$, ..., $c_6(k)$ equal to $c_0(0)$, $c_1(0)$, ..., $c_6(0)$ for $k = 0$. Applying this discussion to the functions $B_{LL}(r,\ t) + 2B_{NN}(r,\ t)$, $B_{p'p'}(r,\ t)$, and $B_{s's'}(r,\ t)$ in particular, and retaining only the principal terms in the right sides, we obtain

$$B_{LL}(r,\ t)+2B_{NN}(r,\ t)\approx 2c_0(0)\frac{\pi^{3/2}}{(2\tilde{\nu}t)^{3/2}}\exp\left(-\frac{r^2}{8\tilde{\nu}t}\right)$$

$$+a_0^2c_3(0)\frac{\pi^{3/2}}{\{[\tilde{\nu}_1+(\gamma-1)(\chi+\chi_{rad})]\}^{3/2}}\exp\left\{-\frac{r^2}{4[\tilde{\nu}_1+(\gamma-1)(\chi+\chi_{rad})]t}\right\},$$

$$B_{p'p'}(r,\ t)=c_3(0)\frac{\pi^{3/2}}{\{[\tilde{\nu}_1+(\gamma-1)(\chi+\chi_{rad})]t\}^{3/2}}\exp\left\{-\frac{r^2}{4[\tilde{\nu}_1+(\gamma-1)(\chi+\chi_{rad})]t}\right\},$$

$$B_{s's'}(r,\ t)\approx c_6(0)\frac{\pi^{3/2}}{[2(\chi+\chi_{rad})t]^{3/2}}\exp\left[-\frac{r^2}{8(\chi+\chi_{rad})t}\right].$$

Here $c_0(0)$, $c_3(0)$, and $c_6(0)$ can be expressed in terms of the known invariants Λ_1, Λ_2, Λ_3 and Λ_4 which have the same meaning as in [6, 9]. In the general case, values of the spectral functions must be given for $t=0$ from some kind of physical considerations.

In conclusion, let us note that for $k_\nu L \gg 1$ the influence of radiation will be identical for any vortex size, while in the case $k_\nu L \ll 1$ the radiation, according to (10), will exert a different influence depending on the size of the turbulent vortices. As follows from the procedure to obtain (10), this fact will hold not only in the gas radiating for $k_\nu L \ll 1$ but also in any media in which there are internal sources of energy liberation and absorption and when this liberated or absorbed energy depends on the temperature or other thermophysical parameters of the medium. For instance, this can be observed in media with chemical reactions.

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